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5. May 2014

Online at <http://mpra.ub.uni-muenchen.de/55728/>

MPRA Paper No. 55728, posted 8. May 2014 07:36 UTC

A Note on Endogenous Growth with Public Capital[□]

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5th May, 2014

Abstract:

This paper develops a two sector model of endogenous economic growth with public capital where private goods and public investment goods are produced with different production technologies. The government buys public investment goods produced by private producers; and the government is a monopsonist in this market. We analyse properties of growth rate maximizing and welfare maximising fiscal policies in the steady state equilibrium. It is shown that the government cannot (can) control the production of public investment good changing the income tax rate (price of public investment good). The growth rate maximizing price of the public investment good is not necessarily equal to its competitive price. However, growth rate maximising income tax rate is equal to the elasticity of private good's output with respect to public capital but is independent of technology in public good production. Welfare maximising solution is not necessarily identical to the growth rate maximising solution even in the steady state equilibrium.

JEL classification: H41; H21; O41

Keywords: Income taxation; Price of public good; Endogenous growth; Steady-state equilibrium; Public capital

[□] This is related to a part of the research work in progress for the Ph.D. degree of Indian Statistical Institute. Helpful and interesting comments from Manash Ranjan Gupta are acknowledged with gratitude.

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1. Introduction

Public capital plays a crucial role on economic growth. Barro (1990) makes the first attempt to incorporate the productive role of public infrastructure in an endogenous growth model; and also analyses the properties of optimal income tax used to finance this productive public expenditure. However, Barro (1990) treats productive public expenditure as a flow variable, rather than a stock variable. Futagami et al. (1993) extends Barro (1990) model considering public capital as a stock variable. After these two models, lots of works have been done in this direction¹. However, in both Barro (1990) and Futagami et al. (1993), both public good and final private good are produced by private firms using identical technology; and then the government buys public goods from private firms using tax revenue and provides freely to private producers as non-rival public input.² In these models, the government chooses optimal tax rate such that the rate of growth and the welfare level are maximized.

This type of modeling has two major problems. First of all, these models assume that the production functions of both goods are identical. In Barro's own words, "*As long as the government and the private sector have the same production functions, the results would be the same if the government buys private inputs and does its own production, instead of purchasing only final output from the private sector, as I assume.*" So it is important to derive the properties of optimal income tax rate where private goods and public goods are produced with different production technologies. Few papers, such as Dasgupta (1999, 2001), Dasgupta and Shimomura (2006), Pintea and Turnovsky (2006), Turnovsky and Pintea (2006) consider different production functions for producing private goods and public goods. However, Pintea and Turnovsky (2006) and Turnovsky and Pintea (2006) do not derive optimal tax rate analytically. On the other hand, Dasgupta (2001) and Dasgupta and Shimomura (2006) do not consider income taxation³. Only Dasgupta (1999) derives optimal tax rate. However, Dasgupta (1999) finds out that the optimal income tax rate is zero and the government should earn entire revenue only by charging the private sector firms for usage of public services on per unit basis. This may be impossible to implement when public services are non-rival and non-excludable in nature; and firms will try to take a free ride. So we stick to the idea of Barro (1990) of freely distributing services of public capital and of charging income taxes to finance its cost.

The second problem with Barro (1990) type of modeling is more severe because it is assumed that the government buys public goods from private producers at the competitive market price. Competitive relative price is equal to unity in the case of identical production

¹ Some examples are Tsoukis and Miller (2003), Petraglia (2003), Clemens (2001), Chen (2003), Hung (2005), Gupta and Barman (2009, 2010, 2013), Barman and Gupta (2010), Agénor (2008), Kitaura (2010), Eicher and Turnovsky (2000), Turnovsky (2000), Ghosh and Roy (2004), Ott and Turnovsky (2006), Ghosh and Gregoriou (2008), Kosempel (2004), Baier and Glomm (2001), Marrero and Novales (2007).

² In Barro's own words, "*But conceptually, it is satisfactory to think of the government as doing no production and owning no capital. Then the government just buys a flow of output (including services of highways, sewers, battleships, etc.) from the private sector. These purchased services, which the government makes available to households, correspond to the input that matters for private production*".

³ Dasgupta and Shimomura (2006) considers lump sum taxes but not per unit income tax.

functions. However, why the government should charge a competitive price is not clear. The government is the only buyer; and so it should act as a monopsonist and should use the relative price as a tool to maximize its objective.

This motivates us to develop the present model. We attempt to analyse the properties of optimal income tax rate used to finance investment in public capital in a two sector economy with different production functions for producing final good and public investment good. In this model, the private sector produces public investment good and sells it to the government who has a monopsony power to set the buying price. Thus this price is also used to control allocation of resources between these two sectors. Otherwise, our model has a framework similar to what Futagami et al. (1993) model has.

We derive many interesting results from this model. First of all, the government can affect inter-sectoral allocation of resources not by changing the income tax rate but by altering the buying price of public investment good. The allocative share of private capital to the production of public investment good varies positively with this price. Secondly, the government's budget balancing income tax rate also varies positively with the buying price of public investment good. Thirdly, growth rate maximising buying price of public investment good is not necessarily equal to its competitive price. Fourthly, the growth rate maximising income tax rate is equal to the elasticity of output with respect to public capital in the production of final private goods only but is independent of the production technology to produce public investment good. Lastly, welfare maximising solutions are different from growth rate maximising solutions even in the steady state growth equilibrium. These results are different from the corresponding results obtained from Barro (1990), Futagami et al. (1993) etc.

Rest of the paper is organized as follows. Section 2 describes the structure of the model. Section 3 deals with the properties of steady state growth equilibrium and growth rate maximizing policies. Section 4 derives properties of the optimal (welfare maximizing) fiscal policies in the steady state equilibrium; and section 5 concludes the paper.

2. The Model

The representative household-producer produces both final good and public investment good using private capital and public capital. Public investment good is defined as the additional stock of non-rival public capital. Production functions of two sectors with different technologies are given by

$$Y = A(\theta K)^\alpha G^{1-\alpha} \quad \text{where } \alpha \in (0,1) \text{ and } A > 0 \quad ; \quad (1)$$

and

$$\dot{G} = B[(1 - \theta)K]^\beta G^{1-\beta} \quad \text{where } \beta \in (0,1) \text{ and } B > 0 \quad . \quad (2)$$

Here, Y , K , G and θ denote amount of final good, stock of private capital, stock of public capital and the share of private capital allocated to final goods sector respectively. \dot{G} represents the amount of public investment good. The government sets the relative price of \dot{G} ; and the household–producer determines the allocation of resources between two sectors. Public capital does not depreciate over time.

The government buys all \dot{G} at the relative price, μ ; and freely provides the whole of G to the household-producers. An income tax at the rate, τ , is charged; and the balanced budget equation is given by

$$\tau Y + \tau \mu \dot{G} = \mu \dot{G} . \quad (3)$$

The representative household is infinitely lived; and she derives instantaneous utility from consumption of final goods only; and maximizes her discounted present value of instantaneous utility subject to her intertemporal budget constraint. The optimization problem is given by the following.

$$\text{Max} \int_0^{\infty} \frac{c^{1-\sigma} - 1}{1-\sigma} e^{-\rho t} dt \quad (4)$$

$$\text{subject to,} \quad \dot{K} = (1-\tau)Y + (1-\tau)\mu\dot{G} - c \quad ; \quad (5)$$

$$K(0) = K_0 \quad ; \quad \theta \in [0, 1]$$

$$\text{and} \quad c \in [0, (1-\tau)Y + (1-\tau)\mu\dot{G}] .$$

Here c is the level of consumption of final goods, and K_0 is historically given initial private capital stock. σ represents the elasticity of marginal utility with respect to consumption and ρ denotes the constant rate of discount. Savings is always invested; and there is no depreciation and consumption of private capital.

Here c and θ are two control variables and K is the only state variable. Solving this dynamic optimisation problem, we obtain⁴

$$(1-\tau)A\alpha\theta^{\alpha-1}K^{\alpha}G^{1-\alpha} = \mu(1-\tau)B\beta(1-\theta)^{\beta-1}K^{\beta}G^{1-\beta} \quad ; \quad (6)$$

and

$$\frac{\dot{c}}{c} = \frac{(1-\tau)A\alpha\theta^{\alpha}K^{\alpha-1}G^{1-\alpha} + \mu(1-\tau)B\beta(1-\theta)^{\beta}K^{\beta-1}G^{1-\beta} - \rho}{\sigma} . \quad (7)$$

Equation (6) shows the efficient allocation of private capital between the two sectors. It implies that the after tax value of marginal product of private capital is same in both the sectors. Equation (6) can be written as

$$\left(\frac{A\alpha}{B\beta}\right) \frac{[(1-\theta)K]^{1-\beta}}{(\theta K)^{1-\alpha}} = \mu G^{\alpha-\beta} . \quad (6a)$$

⁴ Derivation of equations (6) and (7) are shown in the appendix.

Equation (6a) shows that the intersectoral allocation of private capital is independent of the income tax rate, τ , but depends on the government's buying price of public investment good, μ . In Barro (1990), Futagami et al. (1993) and in many other one sector models, the income tax rate, τ , is a tool to determine the level of production. However, in this model, τ does not play any such role but the relative price, μ , can be used as a tool to affect the intersectoral allocation of resources and thus the level of production in the two sectors. From equation (6a) we have

$$\left(\frac{1}{\theta} - 1\right)^{1-\alpha} = \mu \left(\frac{G}{K}\right)^{\alpha-\beta} \frac{B}{A} \frac{\beta}{\alpha} ; \quad (6b)$$

and this equation shows that θ varies inversely with μ given K and G . So as the government raises (lowers) the buying price of public investment good, household-producers allocate more (less) resources for its production. This is so because an increase in μ raises the after tax value of marginal product of private capital in the public good producing sector; and so the share of private capital is increased in that sector. If production functions are identical, i.e., if $A = B$ and $\alpha = \beta$, then

$$\left(\frac{1-\theta}{\theta}\right)^{1-\alpha} = \mu . \quad (6c)$$

So, we can establish the following proposition.

Proposition 1: Government cannot affect intersectoral allocation of private capital by changing the income tax rate but can raise (lower) its allocative share to the public investment good producing sector by charging a higher (lower) relative price of that good.

Equation (7) describes the demand rate of growth consumption which is defined as the excess of marginal return of private capital accumulation over the rate of discount normalized with respect to the elasticity of marginal utility.

Now, from equation (2), we obtain the growth rate of public capital as given by

$$g = \frac{\dot{G}}{G} = B(1-\theta)^\beta \left(\frac{K}{G}\right)^\beta . \quad (8)$$

Using equations (6a) and (8), we obtain

$$\theta = \frac{1}{1 + \frac{B^{\frac{\alpha}{\beta(1-\alpha)}}}{(g)^{\frac{\alpha-\beta}{\beta(1-\alpha)}}} \left(\frac{\mu\beta}{A\alpha}\right)^{\frac{1}{1-\alpha}}} . \quad (9)$$

Equation (9) shows that, if we consider identical production functions in the two sectors, i.e., if $A = B$ and $\alpha = \beta$, then

$$\theta = \frac{1}{1 + \mu^{\frac{1}{1-\alpha}}} . \quad (9a)$$

If $\mu = 1$, then $\theta = 1/2$. So, if the relative price is equal to unity, private capital will be allocated equally between two sectors.

Again, using equations (1), (2), (3), (6a) and (9), we obtain

$$\tau = \frac{\alpha \frac{B^{\frac{\alpha}{\beta(1-\alpha)}}}{(g)^{\frac{\alpha}{\beta(1-\alpha)}} \left(\frac{\mu\beta}{A\alpha} \right)^{\frac{1}{1-\alpha}}}{\alpha \frac{B^{\frac{\alpha}{\beta(1-\alpha)}}}{(g)^{\frac{\alpha}{\beta(1-\alpha)}} \left(\frac{\mu\beta}{A\alpha} \right)^{\frac{1}{1-\alpha}} + \beta} < 1 . \quad (10)$$

Equation (10) shows that the government's budget balancing income tax rate is positive but is less than unity. From equation (10), we have

$$\frac{d\tau}{d\mu} = \frac{\frac{\beta}{1-\alpha} \alpha \frac{B^{\frac{\alpha}{\beta(1-\alpha)}}}{(g)^{\frac{\alpha}{\beta(1-\alpha)}} \left(\frac{\beta}{A\alpha} \right)^{\frac{1}{1-\alpha}} \mu^{\frac{\alpha}{1-\alpha}}}{\left(\alpha \frac{B^{\frac{\alpha}{\beta(1-\alpha)}}}{(g)^{\frac{\alpha}{\beta(1-\alpha)}} \left(\frac{\mu\beta}{A\alpha} \right)^{\frac{1}{1-\alpha}} + \beta \right)^2} > 0 . \quad (11)$$

Equations (11) shows that the government's budget balancing income tax rate varies positively with μ . This is so because an increase in the buying price, μ , raises the government expenditure and so the revenue must rise to balance the budget. However, θ falls and hence Y falls. So the tax rate, τ , must rise to balance the budget. This result is stated in the following proposition.

Proposition 2: The budget balancing income tax rate varies positively with the government's buying price of public investment good.

3. The Steady State Equilibrium

The equations of motion of the system are given by equations (5), (7) and (8). In the steady-state growth equilibrium,

$$g = \frac{\dot{G}}{G} = \frac{\dot{K}}{K} = \frac{\dot{c}}{c} . \quad (12)$$

Now using equations (7), (8), (9), (10) and (12) we have⁵

⁵ Derivation of equation (13) is shown in appendix.

$$\rho + \sigma g = \frac{\beta B^{\frac{1}{\beta}} \mu g^{\frac{\beta-1}{\beta}}}{1 + \alpha \left[\frac{B^{\frac{\alpha}{\beta(1-\alpha)}} \beta^{\frac{\alpha}{1-\alpha}}}{(g)^{\frac{\alpha-\beta}{\beta(1-\alpha)}}} \left(\frac{\mu}{A\alpha} \right)^{\frac{1}{1-\alpha}} \right]} . \quad (13)$$

Equation (13) solves for g and this solution is unique. This equation also shows the nature of the relationship between μ and g .

Now, the government's objective is to maximise the steady-state equilibrium growth rate, g , with respect to μ . We use the first order condition and then obtain the following.⁶

$$\mu = \frac{A(1-\alpha)^{1-\alpha}}{\alpha^{1-2\alpha} B^{\frac{\alpha}{\beta}} g^{\frac{\beta-\alpha}{\beta}} \beta^{\alpha}} . \quad (14)$$

Using equations (13) and (14), we have

$$(\rho + \sigma g^*) g^{*\frac{1-\alpha}{\beta}} = A(1-\alpha)^{1-\alpha} \beta^{1-\alpha} \alpha^{2\alpha} B^{\frac{1-\alpha}{\beta}} . \quad (15)$$

Equation (15) solves for the maximum value of g^* . The left hand side of equation (15) is an increasing function of g^* ; and its right hand side is independent of g^* . So there exists a unique value of g^* .

Putting this value of g^* in equation (14), we obtain⁷

$$\mu^* = \frac{A(1-\alpha)^{1-\alpha}}{\alpha^{1-2\alpha} B^{\frac{\alpha}{\beta}} (g^*)^{\frac{\beta-\alpha}{\beta}} \beta^{\alpha}} . \quad (16)$$

This equation (16) shows that the growth rate maximising μ is not necessarily equal to unity. Even if we consider identical production technology in both the sectors, i.e., $A = B$ and $\alpha = \beta$, then also

$$\mu^* = \left(\frac{1-\alpha}{\alpha} \right)^{1-\alpha} ; \quad (16a)$$

and hence $\mu^* = 1$ if and only if $\alpha = 1/2$, i.e., if and only if production function is symmetric in terms of its arguments. This is stated in the following proposition.

Proposition 3: The steady-state equilibrium growth rate maximising buying price of public investment good is not necessarily equal to its competitive price. The equality is obtained only if production technology in both the sectors are identical and symmetric.

⁶ Derivation of equation (14) is shown in the appendix.

⁷ The second order condition of maximisation of growth rate with respect to μ is satisfied. From equation (13), it can be shown very easily that $\frac{d^2 g}{d^2 \mu} < 0$ when equation (14) holds.

In Barro (1990) and Futagami et al. (1993), this symmetry assumption is not made but the government's buying price of public good is normalized to unity, i.e., to the competitive price with identical technology.

Using equations (16), (9) and (10), we obtain

$$\theta^* = \frac{\alpha^2}{\alpha^2 + \beta(1 - \alpha)} \quad ; \quad (17)$$

and

$$\tau^* = 1 - \alpha \quad . \quad (18)$$

Here θ^* represents the growth rate maximising allocation of private capital to the final goods producing sector in the steady state growth equilibrium. Equation (17) shows that θ^* varies inversely with β and positively with α . This is so because, as β (α) rises, productivity of private capital rises in the public investment good (final good) sector; and, as a result, allocative share of private capital to public investment good (final good) sector goes up. In the case of identical production technology, $\theta^* = \alpha$. So we can establish the next proposition.

Proposition 4: The growth rate maximising allocative share of private capital to final good (public investment good) producing sector varies positively (inversely) with the private capital elasticity of output of final good and varies inversely (positively) with the private capital elasticity of output of public investment good.

Equation (18) leads to the following proposition.

Proposition 5: The steady state equilibrium growth rate maximising income tax rate is equal to the elasticity of output of final good with respect to public capital but is independent of the production technology in the public investment good producing sector.

In Barro (1990) and Futagami et al. (1993), input elasticities of output are same in both the sectors. So this problem does not arise.

4. Welfare Maximization

We use equations (1), (2), (4), (5), (7), (8), (9) and (10) to obtain the welfare level of the representative household, denoted by ω ; and it is given by⁸

⁸ See appendix for derivation of equation (19).

$$\omega = \frac{\left[\frac{\rho}{\alpha} + g \left(\frac{\sigma}{\alpha} - 1 \right) + \frac{(\alpha - \beta) g^{\frac{2\beta-1-\alpha\beta}{\beta(1-\alpha)}} A^{\frac{1}{\alpha-1}} \alpha^{\frac{\alpha-2}{1-\alpha}} \mu^{\frac{2-\alpha}{1-\alpha}} B^{\frac{1}{\beta(1-\alpha)}} \beta^{\frac{2-\alpha}{1-\alpha}}}{\left[1 + \frac{B^{\frac{\alpha}{\beta(1-\alpha)}} (\mu\beta)^{\frac{1}{1-\alpha}}}{(g)^{\frac{\alpha-\beta}{\beta(1-\alpha)}}} \right] \left[\beta + \frac{\alpha B^{\frac{\alpha}{\beta(1-\alpha)}} (\mu\beta)^{\frac{1}{1-\alpha}}}{(g)^{\frac{\alpha-\beta}{\beta(1-\alpha)}}} \right]} \right]^{1-\sigma}}{K_0^{\sigma-1} (1-\sigma) [\rho - g(1-\sigma)]} + \text{constant} \quad (19)$$

If $\sigma > \alpha$ and if $\rho - g(1 - \sigma) > 0$, then equation (19) shows that ω varies positively with g when $\alpha = \beta$. So the growth rate maximising solution is identical to the welfare maximising solution in the steady state equilibrium when $\alpha = \beta$. However, when $\alpha \neq \beta$, i.e., production technologies of two sectors are not identical, then the welfare maximising solution is not identical to the growth rate maximising solution even in the steady state equilibrium. From (19), we obtain⁹

$$\left. \frac{d\omega}{d\mu} \right|_{\mu=\mu^*} = \left\{ \frac{\left[\frac{\rho}{\alpha} + g^* \left(\frac{\sigma}{\alpha} - 1 \right) + \frac{A(\alpha - \beta) g^{*\frac{\alpha-1}{\beta}} B^{\frac{1-\alpha}{\beta}} \beta^{1-\alpha}}{[\alpha^2 + \beta(1-\alpha)](1-\alpha)^{\alpha-2} \alpha^{1-2\alpha}} \right]^{-\sigma}}{K_0^{\sigma-1} [\rho - g^*(1-\sigma)]} \right\} \cdot \left\{ \frac{(\alpha - \beta) g^{*\frac{\beta-1}{\beta}} B^{\frac{1}{\beta}} \alpha^2 \beta}{[\alpha^2 + \beta(1-\alpha)]^2} \right\} \quad (20)$$

If $\sigma > \alpha$ and $\rho > g^*(1 - \sigma)$, then the right hand side of equation (20) is positive (zero) when $\alpha > (=) \beta$. This implies that the welfare maximising value of μ is higher than the growth rate maximising value of μ when the final private good sector is more private capital intensive than the public investment good sector. As a result, the welfare maximising value of income tax rate, τ , exceeds the growth rate maximising value of income tax rate¹⁰. Similarly, the welfare maximising value of θ also exceeds the growth rate maximising value of θ ¹¹.

Barro (1990) and Futagami et al. (1993) show that growth rate maximising income tax rate is identical to the welfare maximising income tax rate in the steady state equilibrium. However, we find that the welfare maximising solution is different from the growth rate maximising solution even in the steady state equilibrium when we consider different production functions for different goods. However two solutions are always identical with identical production technology. So our result generalises the result of Barro (1990) and Futagami et al. (1993). This result is stated in the following proposition.

⁹ See appendix for derivation of equation (20).

¹⁰ Equation (11) shows the positive relationship between τ and μ .

¹¹ Equation (9) shows the inverse relationship between θ and μ .

Proposition 6: Welfare maximising fiscal policies and welfare maximising intersectoral allocation of private capital are different from (identical to) growth rate maximising fiscal policies and growth rate maximising intersectoral allocation respectively in the steady state equilibrium when two sectors have different (identical) production technologies.

5. Conclusions

This paper constructs a simple two sector model with public capital; and derives the properties of optimal fiscal policies in the steady state equilibrium. Both final good and public investment good are produced by the private sector using different production technologies. However, in this model, the government buys public good from private producers at a monopsony price and this buying price is a tool to control allocation of resources between these two sectors. This is how the present model differs from models like Barro (1990), Futagami et al. (1993) etc.

Various interesting findings are obtained here. First, the government can affect intersectoral allocation of private capital not by changing the income tax rate but by altering the buying price of public investment good. This price positively (inversely) affects the share of private capital allocated to the production of public good (final good). Secondly, government's budget balancing income tax rate varies positively with the buying price of public good. Thirdly, growth rate maximising buying price of public good is not necessarily equal to its competitive price. Fourthly, the growth rate maximising income tax rate is equal to the elasticity of output of final good with respect to public capital but is independent of the production technology of public good. At last, welfare maximising solutions are not necessarily identical to growth rate maximising solutions even in the steady state equilibrium.

However, our model is abstract and does not consider many aspects of reality. We do not incorporate the congestion effect of capital accumulation on productivity; and do not consider the role of non-productive public services in the households' utility. Assumption of a benevolent government and exclusion of its political considerations is also a restrictive one. We plan to extend this model in those directions in future.

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Appendix:

Derivation of equations (6) and (7):

Using equations (4) and (5), we construct the Current Value Hamiltonian as given by

$$H_c = \frac{c^{1-\sigma} - 1}{1-\sigma} + \lambda[(1-\tau)Y + (1-\tau)\mu\dot{G} - c] \quad . \quad (A.1)$$

Here λ is the co-state variable. Incorporating equations (1) and (2) in equation (A.1); and then maximising it with respect to c and θ , we obtain following first order conditions.

$$c^{-\sigma} - \lambda = 0 \quad ; \quad (A.2)$$

and

$$\lambda(1-\tau)A(K)^\alpha G^{1-\alpha} \alpha \theta^{\alpha-1} = \lambda \mu(1-\tau)B[K]^\beta G^{1-\beta} \beta(1-\theta)^{\beta-1} \quad . \quad (A.3)$$

From equation (A.3), we obtain equation (6) in the body of the paper.

Again from equation (A.1), we have

$$\frac{\dot{\lambda}}{\lambda} = \rho - (1-\tau)AK^{\alpha-1}G^{1-\alpha} \alpha \theta^\alpha - \mu(1-\tau)BK^{\beta-1}G^{1-\beta} \beta(1-\theta)^\beta \quad ; \quad (A.4)$$

and from equation (A.2), we have

$$\frac{\dot{\lambda}}{\lambda} = -\sigma \frac{\dot{c}}{c} \quad . \quad (A.5)$$

Using equations (A.4) and (A.5), we have equation (7) in the body of the paper.

Derivation of equation (13):

From equation (7), we have

$$\rho + \sigma g = (1-\tau)A\alpha\theta^\alpha \left(\frac{G}{K}\right)^{1-\alpha} + \mu(1-\tau)B\beta(1-\theta)^\beta \left(\frac{G}{K}\right)^{1-\beta} \quad . \quad (A.6)$$

From equation (8), we have

$$\left(\frac{G}{K}\right) = \frac{B^{\frac{1}{\beta}}(1-\theta)}{g^{\frac{1}{\beta}}} \quad . \quad (A.7)$$

From equations (9), (10), (A.6) and (A.7), we obtain equation (13) in the body of the paper.

Derivation of equation (14):

Taking log on both sides of equation (13) and then differentiating it with respect to μ and assuming $\frac{dg}{d\mu} = 0$, we obtain

$$\frac{1}{\mu} = \frac{\frac{\alpha}{1-\alpha} \mu^{\frac{\alpha}{1-\alpha}} \left[\frac{B^{\frac{\alpha}{\beta(1-\alpha)}} \beta^{\frac{\alpha}{1-\alpha}}}{(g)^{\frac{\alpha-\beta}{\beta(1-\alpha)}}} \left(\frac{1}{A\alpha}\right)^{\frac{1}{1-\alpha}} \right]}{1 + \alpha \left[\frac{B^{\frac{\alpha}{\beta(1-\alpha)}} \beta^{\frac{\alpha}{1-\alpha}}}{(g)^{\frac{\alpha-\beta}{\beta(1-\alpha)}}} \left(\frac{\mu}{A\alpha}\right)^{\frac{1}{1-\alpha}} \right]} \quad . \quad (A.8)$$

From equation (A.8), we obtain equation (14) in the body of the paper.

Derivation of equation (19):

From equation (4), we obtain

$$\omega = \frac{c_0^{1-\sigma}}{[\rho - g(1-\sigma)](1-\sigma)} + constant \quad . \quad (A.9)$$

Here, $c(0) = c_0$.

From equation (5), we obtain

$$c_0 = K_0 \left\{ (1-\tau)A(\theta)^\alpha \left(\frac{G_0}{K_0}\right)^{1-\alpha} + (1-\tau)\mu B(1-\theta)^\beta \left(\frac{G_0}{K_0}\right)^{1-\beta} - g \right\} \quad . \quad (A.10)$$

Using equations (7) and (A.10), we obtain

$$c_0 = K_0 \left\{ \frac{\rho + \sigma g}{\alpha} + (1-\tau)\mu B(1-\theta)^\beta \left(\frac{G_0}{K_0}\right)^{1-\beta} \left(\frac{\alpha-\beta}{\alpha}\right) - g \right\} \quad . \quad (A.11)$$

Using equations (8) and (A.11), we obtain

$$c_0 = K_0 \left\{ \frac{\rho + \sigma g}{\alpha} + (1-\tau)\mu B(1-\theta) \left(\frac{B^{\frac{1}{\beta}}}{g^{\frac{1}{\beta}}}\right)^{1-\beta} \left(\frac{\alpha-\beta}{\alpha}\right) - g \right\} \quad . \quad (A.12)$$

Using equations (9), (10) and (A.12), we obtain

$$c_0 = K_0 \left\{ \frac{\rho}{\alpha} + g \left(\frac{\sigma}{\alpha} - 1 \right) + \frac{(\alpha - \beta) g^{\frac{2\beta-1-\alpha\beta}{\beta(1-\alpha)}} A^{\frac{1}{\alpha-1}} \alpha^{\frac{\alpha-2}{1-\alpha}} \mu^{\frac{2-\alpha}{1-\alpha}} B^{\frac{1}{\beta(1-\alpha)}} \beta^{\frac{2-\alpha}{1-\alpha}}}{\left[1 + \frac{B^{\frac{\alpha}{\beta(1-\alpha)}} (\mu\beta)^{\frac{1}{1-\alpha}}}{(g)^{\frac{\alpha-\beta}{\beta(1-\alpha)}} (A\alpha)^{\frac{1}{1-\alpha}}} \right] \left[\beta + \frac{\alpha B^{\frac{\alpha}{\beta(1-\alpha)}} (\mu\beta)^{\frac{1}{1-\alpha}}}{(g)^{\frac{\alpha-\beta}{\beta(1-\alpha)}} (A\alpha)^{\frac{1}{1-\alpha}}} \right]} \right\} \quad (A.13)$$

Using equations (A.9) and (A.13), we obtain equation (19) in the body of the paper.

Derivation of equation (20):

Differentiating equation (19) with respect to μ and evaluating it at $\mu = \mu^*$, we obtain

$$\begin{aligned} & \left. \frac{d\omega}{d\mu} \right|_{\mu=\mu^*} \\ &= \left\{ \frac{\left[\frac{\rho}{\alpha} + g^* \left(\frac{\sigma}{\alpha} - 1 \right) + \frac{(\alpha - \beta) g^{*\frac{2\beta-1-\alpha\beta}{\beta(1-\alpha)}} A^{\frac{1}{\alpha-1}} \alpha^{\frac{\alpha-2}{1-\alpha}} \mu^{*\frac{2-\alpha}{1-\alpha}} B^{\frac{1}{\beta(1-\alpha)}} \beta^{\frac{2-\alpha}{1-\alpha}}}{\left[1 + \frac{B^{\frac{\alpha}{\beta(1-\alpha)}} (\mu^*\beta)^{\frac{1}{1-\alpha}}}{(g^*)^{\frac{\alpha-\beta}{\beta(1-\alpha)}} (A\alpha)^{\frac{1}{1-\alpha}}} \right] \left[\beta + \frac{\alpha B^{\frac{\alpha}{\beta(1-\alpha)}} (\mu^*\beta)^{\frac{1}{1-\alpha}}}{(g^*)^{\frac{\alpha-\beta}{\beta(1-\alpha)}} (A\alpha)^{\frac{1}{1-\alpha}}} \right]} \right]^{\sigma}}{K_0^{\sigma-1} [\rho - g^*(1 - \sigma)]} \right\} \\ & \quad \cdot \left\{ \frac{(\alpha - \beta) g^{*\frac{2\beta-1-\alpha\beta}{\beta(1-\alpha)}} A^{\frac{1}{\alpha-1}} \alpha^{\frac{\alpha-2}{1-\alpha}} \mu^{*\frac{2-\alpha}{1-\alpha}} B^{\frac{1}{\beta(1-\alpha)}} \beta^{\frac{2-\alpha}{1-\alpha}}}{\left[1 + \frac{B^{\frac{\alpha}{\beta(1-\alpha)}} (\mu^*\beta)^{\frac{1}{1-\alpha}}}{(g^*)^{\frac{\alpha-\beta}{\beta(1-\alpha)}} (A\alpha)^{\frac{1}{1-\alpha}}} \right] \left[\beta + \frac{\alpha B^{\frac{\alpha}{\beta(1-\alpha)}} (\mu^*\beta)^{\frac{1}{1-\alpha}}}{(g^*)^{\frac{\alpha-\beta}{\beta(1-\alpha)}} (A\alpha)^{\frac{1}{1-\alpha}}} \right]} \right\} \\ & \quad \cdot \left\{ \left(\frac{2-\alpha}{1-\alpha} \right) \left(\frac{1}{\mu^*} \right) - \frac{\frac{\mu^{*\frac{\alpha}{1-\alpha}} B^{\frac{\alpha}{\beta(1-\alpha)}} (\beta)^{\frac{1}{1-\alpha}}}{1-\alpha} \frac{1}{(g^*)^{\frac{\alpha-\beta}{\beta(1-\alpha)}} (A\alpha)^{\frac{1}{1-\alpha}}}}{\left[1 + \frac{B^{\frac{\alpha}{\beta(1-\alpha)}} (\mu^*\beta)^{\frac{1}{1-\alpha}}}{(g^*)^{\frac{\alpha-\beta}{\beta(1-\alpha)}} (A\alpha)^{\frac{1}{1-\alpha}}} \right]} - \frac{\frac{\mu^{*\frac{\alpha}{1-\alpha}} \alpha B^{\frac{\alpha}{\beta(1-\alpha)}} (\beta)^{\frac{1}{1-\alpha}}}{1-\alpha} \frac{1}{(g^*)^{\frac{\alpha-\beta}{\beta(1-\alpha)}} (A\alpha)^{\frac{1}{1-\alpha}}}}{\left[\beta + \frac{\alpha B^{\frac{\alpha}{\beta(1-\alpha)}} (\mu^*\beta)^{\frac{1}{1-\alpha}}}{(g^*)^{\frac{\alpha-\beta}{\beta(1-\alpha)}} (A\alpha)^{\frac{1}{1-\alpha}}} \right]} \right\} \quad (A.14) \end{aligned}$$

Now, from equations (9) and (10), we find that the last bracket term is equal to $\left(\frac{1}{\mu^*}\right)\left\{\frac{2-\alpha}{1-\alpha} - \frac{1}{1-\alpha}[(1-\theta^*) + \tau^*]\right\}$. Again, from equation (9), it appears that $\left[1 + \frac{\frac{\alpha}{B\beta^{(1-\alpha)}}}{(g^*)^{\frac{\alpha-\beta}{\beta(1-\alpha)}}}\left(\frac{\mu^*\beta}{A\alpha}\right)^{\frac{1}{1-\alpha}}\right]$ is equal to $\left(\frac{1}{\theta^*}\right)$; and from equations (9) and (10), we find that $\left[\beta + \frac{\frac{\alpha}{B\beta^{(1-\alpha)}}}{(g^*)^{\frac{\alpha-\beta}{\beta(1-\alpha)}}}\left(\frac{\mu^*\beta}{A\alpha}\right)^{\frac{1}{1-\alpha}}\right]$ is equal to $\frac{\alpha(1-\theta^*)}{\theta^*\tau^*}$. Incorporating all these equalities and putting values of μ^* , τ^* and θ^* from equations (16), (17) and (18), we obtain equation (20).